Demography General

Useful Formulae

Spring 2017

Population growth:

$$N(T) = N(0)e^{\int_0^T r(t)dt},$$

$$N(T) = N(0)e^{rT}, \quad r = \log\left(\frac{N(T)}{N(0)}\right)/T$$

Hazard and survival:

$$\mu(x) = \frac{d(x)}{l(x)}, \qquad l(x) = l(0)e^{-\int_0^x \mu(a)da},$$

$$e(x) = \int_x^\infty l(a)da/l(x)$$

Rates to probabilities:

$$_nq_x = \frac{n\,{}_nm_x}{1 + (n -_n a_x)\,{}_nm_x}, \qquad {}_nq_x = 1 - e^{-n\,{}_nm_x}$$

Time lived:

$$_nL_x = n\,l_{x+n} + {}_na_x\,{}_nd_x, \qquad {}_nL_x = \frac{l_x - l_{x+n}}{{}_nm_x}, \qquad {}_\infty L_x = \frac{l_x}{{}_\infty m_x}$$

Gompertz:

$$\log \mu(x) = \alpha + \beta x$$

Brass:

$$Y_x = \alpha + \beta Y_x^s \quad \text{where} \quad Y_x = 0.5\log\frac{l_0 - l_x}{l_x} \quad \text{so} \quad l_x = l_0\frac{1}{1 + e^{2Y_x}}$$

Unobserved heterogeneity:

$$\mu(x|\theta) = \mu_0(x)\theta, \qquad \mu(x) = \mu_0(x)E(\theta|X > x)$$

Lee-Carter:

$$\log\,{}_nM_{x,t} = a_x + b_x\,k_t$$

Coale-McNeil:

$$G(a) = cGs(\frac{a - a_0}{k}) = cG_0(\frac{a - \mu}{\sigma})$$

Hernes:

$$g(a) = Ae^{-ra}G(a)[1 - G(a)]$$

Coale and Coale-Trussell:

$$r(a) = Mn(a)e^{-mv(a)}, v(a) \geq 0 \quad \text{and} \quad f(a) = G(a)r(a)$$

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$$r(a, d) = n(a)e^{\alpha + \beta d}$$

Bongaarts:

$$\mathrm{TFR} = \mathrm{TN}\, C_m\, C_c\, C_i\, C_a$$

Leslie matrix:

$$\frac{{}_nL_0}{l_0}({}_nF_x + \frac{{}_nL_{x+n}}{{}_nL_x}\, {}_nF_{x+n})/2$$

$$\frac{{}_nL_{x+n}}{{}_nL_x}, \qquad \frac{T_{x+n}}{T_x}$$

Net reproduction rate:

$$\mathrm{NRR} = \int_\alpha^\beta p(a)m(a)da \approx \mathrm{GRR}\, p(A_M)$$

Stationary population:

$$be_0 = 1, \qquad c(a) = p(a)/e_0$$

Renewal equation:

$$B(t) = \int_\alpha^\beta B(t-a)p(a)m(a)da, t > \beta$$

Lotka's equation:

$$1 = \int_\alpha^\beta e^{-ra}p(a)m(a)da$$

Stable age distribution:

$$c(a) = be^{-ra}p(a)$$

"Sheer Poetry":

$$\frac{d \log c(a)}{dr} = A_P - a, \quad \text{where} \quad A_p = \int ac(a)da/\int c(a)da$$

Stable population birth rate:

$$b = 1/\int_0^\infty e^{-ra}p(a)da$$

Mean length of a generation (T):

$$r = \frac{\log(\mathrm{NRR})}{T}, \qquad T \approx \frac{A_B + \mu}{2}$$

Population momentum

$$M = \int_0^\beta \frac{c(a)}{c_s(a)}w(a)da \quad \text{(P-G)} \qquad M = \frac{be_0}{\sqrt{\mathrm{NRR}}} \quad \text{(K-F)}$$

Tempo-adjustment:

$$\mathrm{TFR}^* = \frac{\mathrm{TFR}}{1-r}$$