

Stationary Populations

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We discussed the interpretation of the life table as a stationary population. The following example illustrates some of the ideas.

A Social Planning Application

The population of a certain country is stationary, following the abridged life table below, which for simplicity we assume applies to both males and females. There are 10,000 births per year and no migration.

Age x	l_x	${}_n d_x$	${}_n q_x$	${}_n L_x$	T_x	e_x
0	100000	1147	0.01147	99197	7123096	71.23
1	98853	264	0.00267	394798	7023899	71.05
5	98589	164	0.00166	492511	6629101	67.24
10	98425	160	0.00163	491756	6136590	62.35
15	98265	604	0.00615	490137	5644834	57.45
20	97661	789	0.00808	486262	5154697	52.78
25	96872	665	0.00686	482658	4668436	48.19
30	96207	609	0.00633	479525	4185778	43.51
35	95598	778	0.00814	476170	3706254	38.77
40	94820	1269	0.01338	471198	3230084	34.07
45	93551	2154	0.02302	462814	2758886	29.49
50	91397	3528	0.03860	448818	2296072	25.12
55	87869	5434	0.06184	426642	1847254	21.02
60	82435	8030	0.09741	393303	1420612	17.23
65	74405	11342	0.15244	345084	1027309	13.81
70	63063	14656	0.23240	279824	682225	10.82
75	48407	16383	0.33844	201217	402401	8.31
80+	32024	32024	1.00000	201185	201185	6.28

School education is mandatory between ages 5 and 15. A presidential candidate proposes a social solidarity plan that includes an annual allowance of \$2,000 for each child under the minimum school leaving age, a housing grant of \$5,000 in cash to each couple on their first marriage, a cash grant of \$400 for deaths under age 10 and \$800 for deaths at age 10 and above, and an old age pension of \$6,000 per year to each person over age 65.

- What's the total population of the country?
- If all citizens marry for the first time at age 25, what's the annual number of marriages?
- What's the total number of children under minimum school living age?

- (d) What's the annual number of deaths under age 10? At ages 10 and over?
- (e) What's the number of old age pensioners?
- (f) How much would the candidate's plan cost per year?
- (g) Which benefit is the most expensive?
- (h) If three-quarters of the population between ages 15 and 25 and half of the population between ages 25 and 65 are employed in the labor force, how much tax would each have to pay to support the social solidarity plan entirely out of direct taxation?

There are only 10,000 births per year, so we need to adjust the radix of the life table to match. For simplicity I assume below that this is the case, applying a "reduction factor" of 0.1 to all columns representing counts ($l_x, {}_n d_x, {}_n L_x, T_x$)

- a) The total population is $T_0 = 712,310$. (Not 7 million.)
- b) The number of people reaching age 25 each year is l_{25} but the number of marriages is half that because it takes two to Tango: $\frac{1}{2}l_{25} = 4,844$
- c) The number of children under age 15 is ${}_{15}L_0 = T_0 - T_{15} = 147,826$.
- d) The annual number of deaths under age 10 is ${}_{10}d_0 = l_0 - l_{10} = 157.5$. The number of deaths at ages 10 or older is ${}_{\infty}d_{10} = l_{10} = 9,842.5$. These add up to 10,000. (You may round if you wish.)
- e) The number of old age pensioners or people at ages 65 and above is $T_{65} = 102,731$.
- f) Here is a quick annual budget to go with the campaign promises:

Item	Calculation	Cost (\$1,000's)
Child allowance	$\$2,000 \times 147,826$	295,652
Housing grant	$\$5,000 \times 4,844$	24,218
Death grant	$\$400 \times 157.5 + \$800 \times 9,842.5$	7,937
Old age pension	$\$6,000 \times 102,731$	616,385
Total		944,193

The total cost is just shy of a billion dollars. (Total w/o rounding is \$944,192,800.)

- g) The most expensive benefit is the old age pension, by far.
- h) The population at ages 15 to 25 is $T_{15} - T_{25} = 97,640$ and at ages 25 to 65 is $T_{25} - T_{65} = 364,113$. Given the proportions in the labor force (75% and 50%) we have 255,286 taxpayers ($0.75 \times 97,640 + 0.50 \times 364,113 = 255,286$). The assessment need to balance the books is \$3,699 per person employed ($\$944,192,800 / 255,286.2 = \$3,698.57$).

This problem is adapted from A. H. Pollard, F. Yusuf and G. N. Pollard (1990) *Demographic Techniques*. Third Edition. Sydney: Pergamon Press.