Fertility and Reproduction

This topic is covered in some detail in Chapter 5 of the textbook. Here is a summary of the main ideas. (Separate notes deal with birth intervals and the proximate determinants of fertility.)

Period Fertility

Make sure you are familiar with the standard period measures, the Crude Birth Rate (CBR) or births per 1000 population (or person-years), the General Fertility Rate (GFR) or births per 1000 woman-years in the reproductive ages (usually 15-44), and the Total Fertility Rate (TFR) equal to the sum of single-year age-specific fertility rates (or five times the sum of 5-year age-specific fertility rates) divided by 1,000.

The TFR is interpreted as the average number of children a (synthetic) cohort of women would have if it went through life having children at the current age-specific fertility rates. It can also be interpreted as a standardized GFR, where the standard age distribution is uniform (has the same number of women at every age). The actual GFR uses the age distribution as weights.

The Total Marital Fertility Rate (TMFR) is constructed in a similar fashion using age-specific marital fertility rates, or marital births per 1000 married woman-years in each age group. It represents the expected number of children a woman would have if she married at 15 and stayed married through the reproductive years having children at current marital fertility rates. A much better measure would use duration of marriage rather than age as the clock, but the requisite data are rarely available.

The Princeton Fertility Indices

In his study of fertility in historical European population Coale wanted to compare fertility rates adjusted for age, but the required age-specific fertility rates were not always available. He did, however, have the age distributions. So he adopted an indirect approach. He used as standard a set of rates from the Hutterites, a group with very high fertility, and computed an index of fertility defined as the ratio of observed to expected births if all women had children at Hutterite rates:

$$I_f = \frac{B}{\sum H_i W_i} = \frac{\sum F_i W_i}{\sum H_i \, W_i}$$

where B is the number of births, F_i stands for age-specific fertility rates (with H_i for the Hutterite rates) and W_i is the number of women in the age group. The analogy with the SMR should not go unnoticed.

Assuming that births occur only within marriage, Coale was able to decompose I_f into an index of marriage I_m and an index of marital fertility I_g, such that

$$I_f = I_m I_g$$

The index of marital fertility is defined in a form analogous to the index of general fertility as

$$I_g = \frac{\sum F_i^L W_i^L}{\sum H_i W_i^L} \text{ or } \frac{B}{\sum H_i W_i^L}$$

where F_i^L is the age-specific marital fertility rate and W_i^L is the number of married women in that age group (the L stands for legitimate). If there is no extra-marital fertility $B = B^L = \sum F_i^L W_i^L$, the numerator of both I_g and I_f. The index of marriage is defined as

$$I_m = \frac{\sum H_i W_i^L}{\sum H_i W_i}$$

and is essentially a weighted average of proportions married by age using the Hutterite fertility rates as weights. (Notation might be clearer if we used M_i instead of W_i^L for married women in age group i.)

Box 5.2 in the textbook illustrates the calculations for the French village of Tourouvre-au-Perche in 1801, for which $I_f = 0.364$, $I_g = 0.70$ and $I_m = 0.52$. Fertility was 36.4% of what it would be if all women had births at Hutterite rates. This was due in part to lower marital fertility, as births were only 70% what they would be if married women had children at Hutterite rates. The main explanation, however, lies in the proportions married by age, which amount to only 52% when weighted by Hutterite rates. (The fertility decline in Europe turned out to be driven largely by marriage delays.)

Reproduction

The term reproduction is used to denote single-sex fertility rates, usually female births to women. The corresponding age-specific rates are called *maternity* rates, and are computed with only female births in the numerator. The textbook uses ${}_nF_x^F$ to denote the maternity rate for ages x to $x+n$, and $m(a)$ to denote the maternity function in continuous time.

The Gross Reproduction Rate (GRR) is the female equivalent of the TFR, essentially a sum of age-specific *maternity* rates, which is interpreted as the average number of daughters a woman would have if she went through the reproductive span having daughters at current maternity rates. In continuous time, if the reproductive years go from ages α to β the GRR is

$$GRR = \int_\alpha^\beta m(a)da$$

A better measure of reproduction is the Net Reproduction Rate (NRR), which multiplies each maternity rate ${}_nF_x^F$ by the probability of surviving to that age, using ${}_nL_x^F/nl_0$ from an appropriate female life table. We interpret NRR as the average number of daughters a newborn woman would have if she was subject to the observed survival probabilities and maternity rates. In continuous time

$$NRR = \int_\alpha^\beta p(a)m(a)da$$

where $p(a) = l(a)/l(0)$ is the probability of surviving from birth to exact age a.

Obviously the NRR must be one for the female population to replace itself, in which case we say that fertility is at *replacement level*. (This doesn't say anything about the age pattern of fertility, as many schedules can lead to replacement level.) We'll now see what this level implies in terms of more common measures, namely the GRR and the TFR.

If every woman survived from birth to the end of the reproductive years the GRR and NRR would be equal. Coale has shown that to a good approximation

$$NRR = p(A_M)GRR$$

where A_M is the mean age of the maternity schedule, a weighted average using the maternity function as the weights:

$$A_M = \frac{\int a\, m(a)da}{\int m(a)da}$$

In practice A_M is computed as a weighted sum, working with the midpoints of the age groups $x + \frac{n}{2}$ and with weights given by the maternity rates ${}_nF_x^F$.

Using this approximation, replacement level fertility corresponds to $GRR = 1/p(A_M)$. If the sex ratio at birth does not depend on the age of the mother and there are 2.05 total births for each female birth, then $TFR = 2.05\ GFR$ and the replacement level TFR is approximately $2.05/p(A_M)$. In most developed countries $p(A_M)$ is pretty close to one and the replacement level TFR is about 2.1. In high mortality countries many women die before reaching the mean age of childbearing and the replacement level TFR can be much higher. Espenshade, Guzmán and Westoff noted surprising global variation in replacement fertility, ranging from less than 2.1 to nearly 3.5.

Box 5.5 in the textbook shows that in the U.S. in 1991 the GRR was 1.013 daughters per woman and the NRR was 0.995, so fertility was below replacement level. The fact that the two indices are so similar indicates a fairly high probability of surviving to the mean age of childbearing, in fact about 98.2%. The mean age of the maternity schedule is not shown in the textbook, but can easily be verified to be 26.5.

A final note of caution: an $NRR > 1$ indicates that the population is growing, as each cohort of women leaves behind a larger cohort of daughters, but it doesn't tell us how fast it is growing. As we'll see, this also depends on when they have their daughters, or more precisely on the mean length of a generation.

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